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Derivation of the Mass of the Observable Universe

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Using solely microphysical arguments, we arrive at an expression for the overall mass of the universe in terms of the constants of nature and the parameters describing the elementary particles. The link between cosmology and the microscopic world is thus highlighted in the spirit of Dirac's large number hypothesis.

The empirical relations between microscopic and macroscopic quantities are the basis of Dirac's (1937) large number hypothesis (LNH). The large dimensionless ratio between the Hubble time and the atomic time and the ratio of the electromagnetic to the gravitational interaction between two elementary particles are of the same order, $O(10^{40})$. According the LNH this is not coincidental, but must express a deep theoretical meaning. A consequence of this is a cosmology in which one or more physical constants may vary with time. This connection between the universe as a whole and the atomic world is poorly understood. Of course, any hint on how this comes about is of great interest to our understanding of the Dirac proposition. Here, our aim is to derive the mass of the observable universe using just microscopic quantities in order to highlight Dirac's idea of the link between cosmology and microphysics.

In the Friedmann model of the expanding universe, its present density ρ_o is related to the expansion rate H_o through (Weinberg, 1972)

$$G\rho_o \simeq H_o^2$$

Thus, the total mass M_u contained in the observable universe can be estimated by taking $M_u \sim \rho_o R_o^3$, where $R_o \sim cH_o^{-1}$ is its size. Substituting ρ_o, we obtain

$$M_u = \frac{c^3}{GH_o} \tag{1}$$

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which is the expression we shall derive below following a different path.

There has been some attempts to calculate the mass of the universe solely from microphysical considerations. Sivaram (1982) gives an expression for it in terms of the coupling constants of the elementary particle interactions, that is, gravitational, strong, weak, and electromagnetic interactions. In what follows, we give a very simple argument from the realm of microphysics to arrive at an expression for the mass of the universe identical to (1).

The present size of the universe is so large that the force which dominates the overall interaction between its hadronic masses is the gravitational force. In the distant past, however, when the density of the universe was very much higher than it is today, matter was so compactified that particles were at atomic distances from one another.

Let us suppose that we can bring together all particles in the universe to a distance of the order of the range of the strong interaction. The density would then be so high that quantum effects would become important and the classical field theory of gravitation would no longer be valid. For instance, a quantum description of gravitational fields will be required when the size R of the universe is of the same order as the wavelength of a typical elementary particle wave function. In the case of a π meson, this is of the order of the range of the strong interaction and therefore

$$R \sim l_\pi = \frac{\hbar}{m_\pi c} \tag{2}$$

At this stage, when the quantum-gravity effect builds up, the density ought to be Planck's density, that is,

$$\rho = \frac{c^5}{G^2 \hbar} \tag{3}$$

If we now compute the total mass M inside the volume defined by R, i.e., ρR^3, we obtain, using (2) and (3),

$$M = \frac{c^2 \hbar^2}{G^2 m_\pi^3} \tag{4}$$

By substituting the numerical values of the quantities in the above expression we get

$$M \sim 1.6 \times 10^{55}\ g$$

which is of the order of the mass of the visible universe.

At this point, we introduce the interesting empirical relation given by Weinberg (1972) which relates the mass of a typical elementary particle, in this case the pion, and other microscopic and macroscopic constants, namely

$$m_\pi \simeq \left(\frac{\hbar^2 H_o}{Gc}\right)^{1/3} \tag{5}$$

If we substitute this into (4), we obtain an expression for M in terms of the Hubble parameter, G, and c. By doing so, we finally get

$$M = \frac{c^3}{GH_o} \tag{6}$$

This is identical to expression (1) derived in the context of Friedmann's cosmological model.

We can use the Planck mass $m_{\rm Pl} = (\hbar c/G)^{1/2}$ to rewrite equation (4) in a different form,

$$M = \left(\frac{m_{\rm Pl}}{m_\pi}\right)^3 m_{\rm Pl} \tag{7}$$

and give it the following interpretation. At the epoch we are considering, when the density is $\rho_{\rm Pl}$, the horizon is the Planck length $l_{\rm Pl} \propto 1/m_{\rm Pl}$, while the overall size of the universe is $R \propto 1/m_\pi$ [equation (2)]. Thus, we have

$$\left(\frac{m_{\rm Pl}}{m_\pi}\right)^3 = \left(\frac{R}{l_{\rm Pl}}\right)^3 \tag{8}$$

which can be interpreted as the number N of causally disconnected regions in the universe, each of these regions having a mass $m_{\rm Pl}$. Therefore, equations (7) and (8) give for the total mass of the universe

$$M = Nm_{\rm Pl}$$

It is worth mentioning that $N \sim 10^{60}$ is a power of Dirac's large dimensionless number $(10^{40})^{3/2}$.

We have seen that, from a very simple argument, a new derivation of the mass of the universe is possible which involves only microscopic quantities. This strengthens the idea that there must exist a deep relation between the microscopic and the macroscopic world within a complete and unified theory of the forces of nature.

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